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FIRST YEAR FINAL REPORT

SOLAR FLARE PHYSICS

November 2, 2001

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Solar Flare Physics

During the past year we have been working with the HESSI (High Energy Solar Spectroscopic Imager team in preparation for launch in early 2002. HESSI has as its primary scientific goal photometric imaging and spectroscopy of solar flares in hard X-rays and gamma-rays with $\sim 2''$ angular resolution, ~keV energy resolution and ~2-s time resolution over the 6-keV to 15-MeV energy range.

To help improve the software used for imaging with the HESSI telescope, we have conducted studeies on several different topics. Among these are:

I A New Method for Obtaining Visibilities from Modulation Data

II HESSI Test-data Reconstructions

III Measures of HESSI Phase Diversity

IV Over Resolution Evaluation

The text and figures for these are attached.

This work will continue into the HESSI launch and early mission phase. Shortly after launch, the effort will shift into imaging of solar flares and performing self-calibrations using methods derived by the PI for this grant and by other members of the HESSI team.

I. A New Method for Obtaining Visibilities from HESSI Modulation Data (April 2001)

A New Method for reconstructing Visibilities

When an RMC does not contain double-pair *sine* and *cosine* subcollimators, but only single-pair subcollimators (as in *HESSI*), one must derive a second profile from the single one. That is, if we identify the the given modulation profile with the *cosine* component (which corresponds to the real part of the Visibility) then we must find the *sine* component (which is the imaginary part of the Visibility).

One of the ways to do this involves fitting sinusoids to each cycle of the modulation profile, and shifting them by 90°, and this is done by hsi_calib_ev2vis.pro in the *HESSI*) SSW library. However, this program has certain drawbacks of its own, and it is important to create other methods which may mitigate these drawbacks. (Or at least move the perhaps inevitable failings to a different region of parameter space)..

A new method for reconstructing visibilities from modulation profiles is based on a surprising relationship between the Fourier transforms of the real and imaginary parts of the visibility due to a point source. For a point source, the Fourier components (I_n) of the imaginary part of the visibility are simple linear combinations of the two neighboring Fourier components (R_n) of the real part of the visibility:

$$I_n = a_n * R_{n-1} + b_n * R_{n+1}$$

Since an arbitrary source may be represented as a superposition of point sources, if the source is sufficiently compact (i.e. its extent is much less than the distance to the spin axis), the linear relationship still holds.

The plan of the new method is as follows:

- Perform an FFT on a suitably normalized count profile;
- Apply the linear filter derived from the above equation;
- Perform an inverse FFT, and return the imaginary part of the visibility.

The mathematics of this new technique is contained in the .ps or .pdf files:
new_vis_method.ps
new_vis_method.pdf

Ed Schmahl
Last modified: Thu Apr 12, 2001

1 Visibility Harmonics

We start off with the simple case of a point source at a distance a from the spin axis (which is assumed for now to be steady and at the center of the coordinate system). The polar coordinates of the visibility plane are (k, ϕ). The point-source visibility is:

$$V = A\, e^{ika\ cos\phi}$$
$$= A\, e^{ika\frac{z+z^{-1}}{2}}, \qquad \text{where } z = e^{i\phi}$$

$$= A\,[1\ +\ ika\frac{z+z^{-1}}{2}\ -\ \frac{(ka)^2}{2!}\ (\frac{z+z^{-1}}{2})^2 +\ \cdots]$$

$$= A\ \sum_{m=-\infty}^{\infty} J_m(ka)(iz)^m, \qquad \text{Abramowitz and Stegun, 1946}$$

Now we expand the exponential z into sines and cosines, and then write out the lowest-order terms explicitly:

$$V = A\ \sum_{m=-\infty}^{\infty} J_m(ka)\ i^m\ [cos(m\phi) + i\ sin(m\phi)], \qquad \phi = \phi - \psi$$

$$= A\ [J_0(ka) + J_{-1}(ka)i^{-1}(cos\phi - isin\phi) +\ J_1(ka)i(cos\phi + isin\phi) +$$
$$J_{-2}(ka)i^{-2}(cos2\phi - isin2\phi) +\ J_2(ka)i^2(cos2\phi + isin2\phi) +$$
$$J_{-3}(ka)i^{-3}(cos3\phi - isin3\phi) +\ J_3(ka)i^3(cos3\phi + isin3\phi) +$$
$$J_{-4}(ka)i^{-4}(cos4\phi - isin4\phi) +\ J_4(ka)i^4(cos4\phi + isin4\phi) + \cdots]$$

$$= A\ [J_0(ka) + iJ_1(ka)(cos\phi - isin\phi) +\ iJ_1(ka)(cos\phi + isin\phi) +$$
$$-J_2(ka)(cos2\phi - isin2\phi) +\ -J_2(ka)(cos2\phi + isin2\phi) +$$
$$-iJ_3(ka)(cos3\phi - isin3\phi) +\ -iJ_3(ka)(cos3\phi + isin3\phi) +$$
$$J_4(ka)(cos4\phi - isin4\phi) +\ J_4(ka)(cos4\phi + isin4\phi) + \cdots]$$

$$= A\,[J_0(ka) + 2iJ_1(ka)cos\phi - 2J_2(ka)cos2\phi - 2iJ_3(ka)cos3\phi + 2J_4(ka)cos4\phi + \cdots]$$

Now we can look at the Real and Imaginary parts of V separately, to see if there are any hidden relationships:

$$Re(V) = A\,cos(ka\,cos\phi) = A\,[J_0(ka) - 2J_2(ka)cos2\phi + 2J_4(ka)cos4\phi + \ldots]$$

$$Im(V) = A\,sin(ka\,cos\phi) = 2A\,[J_1(ka)cos\phi\ -\ J_3(ka)cos3\phi\ +\ \cdots]$$

The last two equations show that the coefficients of the Im(V) series must be related to the neighboring coeffcients of the Re(V) series because of the recurrence relation for Bessel functions.

$$J_m(z) = (z/2m)[J_{m-1}(z) + J_{m+1}(z)]$$

Since the visibility is a linear superposition of visibilities of point sources, it may be possible to exploit this recurrence formula to obtain the imaginary part of the visibility from the real part.
To clarify, we rewrite the equations in the form:

$$Re(V) = \sum_{-\infty}^{\infty} a_m cos(m\phi)$$

$$= A\,[\cdots + J_{-4}cos(-4\phi) - J_{-2}cos(-4\phi) + J_0cos(0\phi) - J_2cos(2\phi) + J_4cos(4\phi) + \cdots]$$

$$Im(V) = \sum_{-\infty}^{\infty} b_m cos(m\phi)$$

$$A\,[\cdots - J_{-5}cos(-5\phi) + J_{-3}cos(-3\phi) - J_{-1}cos(-\phi) + J_1cos(\phi) - J_3cos(3\phi) + \cdots]$$

Comparing the coefficients with the recurrence relation, we find that:

$$b_m = (ka/2m) * (a_{m-1} - a_{m+1})$$

Although ka may be unknown, this equation shows that at least the profile of the imaginary coefficients b_m can be found from the a_m.

1.1 Source at Arbitary Azimuth

The case we just explored was very special: the source lay on the X axis, so there were no $sin(m\phi)$ terms in the Fourier expansions. Now we look at the case where the source is located at (a, ψ). The equations are very similar to the ones above, except that ϕ is replaced with $\phi - \psi$:

$$V = A\ e^{ika\ cos(\phi-\psi)}, \qquad z = e^{i(\phi-\psi)}$$

$$= A \sum_{m=-\infty}^{\infty} J_m(ka)\ i^m\ [cos(m(\phi-\psi) + i\ sin(m(\phi-\psi))]$$

So all the expansions remain the same with ϕ replaced by $\phi - \psi$
Now, however we come to a complication. Instead of a_m, we are given the Fourier coefficients $a_m e^{-im\psi}$ and $a_m e^{im\psi}$, where ψ is unknown:

$$Re(V) = \sum_{-\infty}^{\infty} a_m cos(m(\phi-\psi)) = \frac{1}{2}\sum_{-\infty}^{\infty}\{a_m e^{-im\psi}\ e^{im\phi} + a_m e^{im\psi}\ e^{-im\phi}\}$$

This would be a trivial problem, since the product of the two terms is a real positive number, a_m^2, except that the sign of a_m must be determined and ψ must be independent of m. If we force a_m to be positive, then we put the onus for the sign on $e^{-im\psi}$, and this can make ψ be different for different m, which is not allowed.

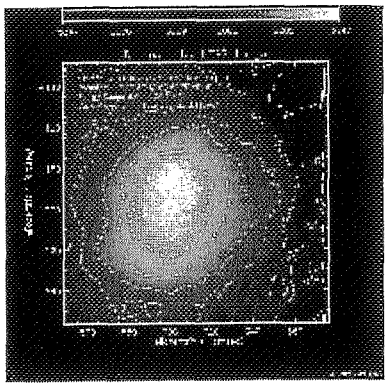
Once the a_m and ψ have been found, the same recurrence relations hold, and one may determine Im(V) from Re(V).

1.2 Superposed Sources

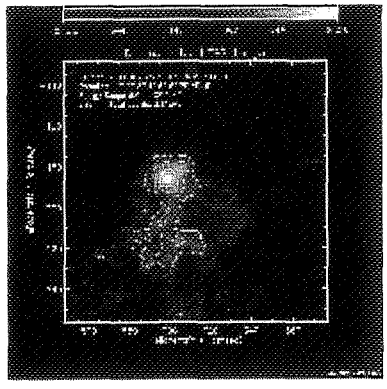
Some tests using IDL have been made with double sources, and it appears that the recurrence relation that relates the real and imaginary parts still works to a good approximation.

II. HESSI Test-data Reconstructions (May 2001)

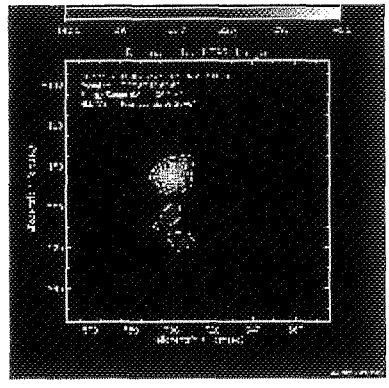
HESSI TEST-DATA Reconstructions



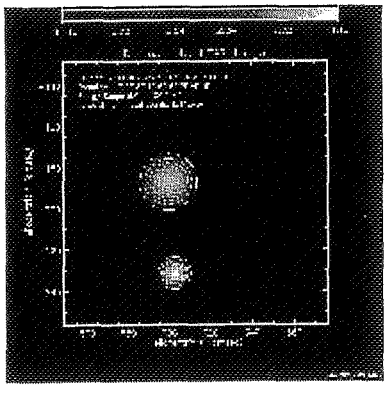
Flare 6 BACK PROJECTION



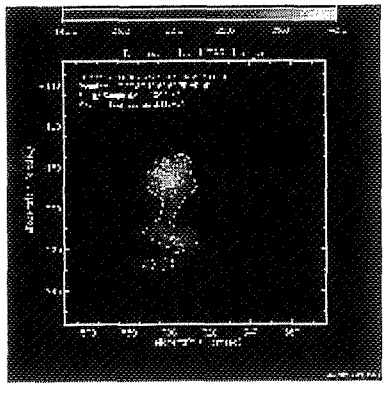
Flare 6 CLEAN



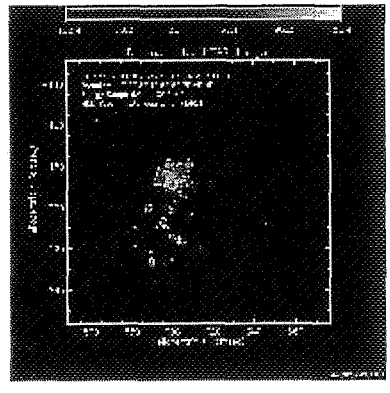
Flare 6 MEM_VIS



Flare 6 FORWARD FIT



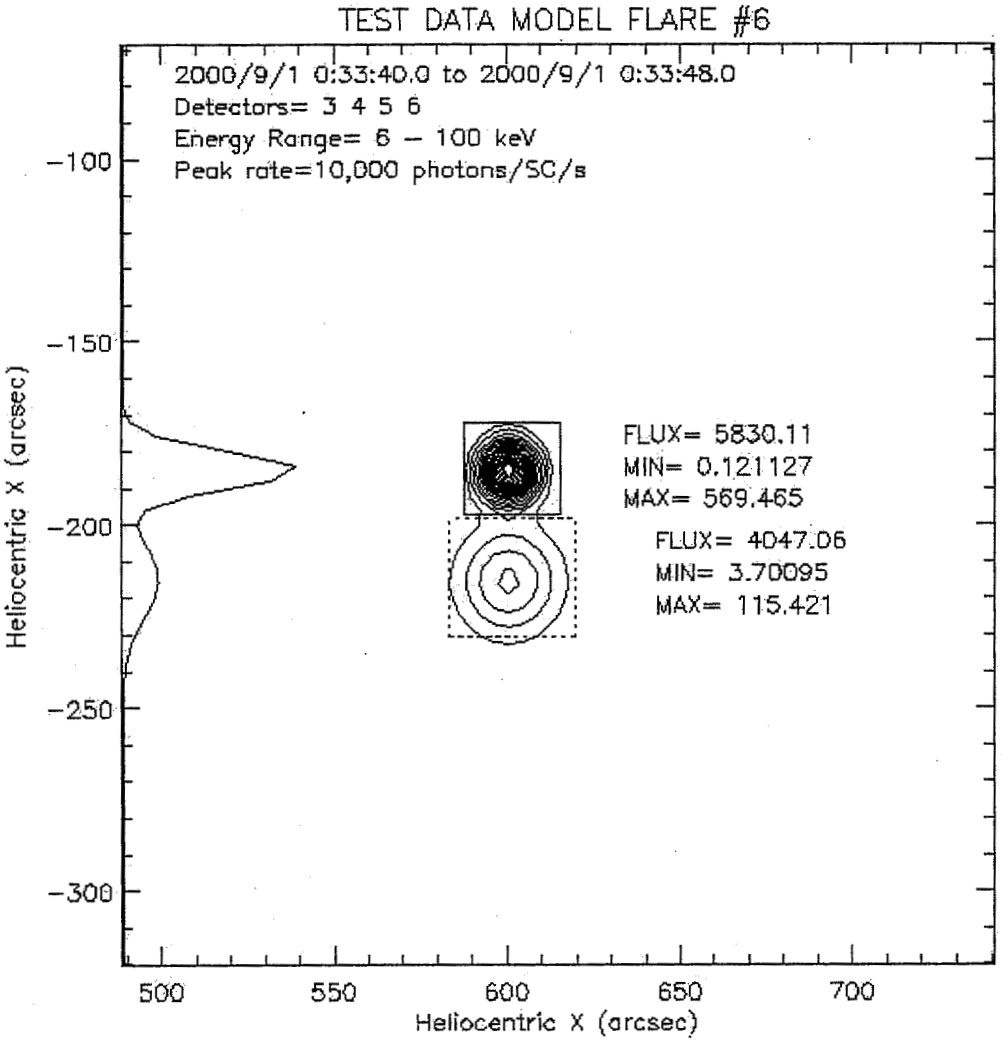
Flare 6 PIXONS



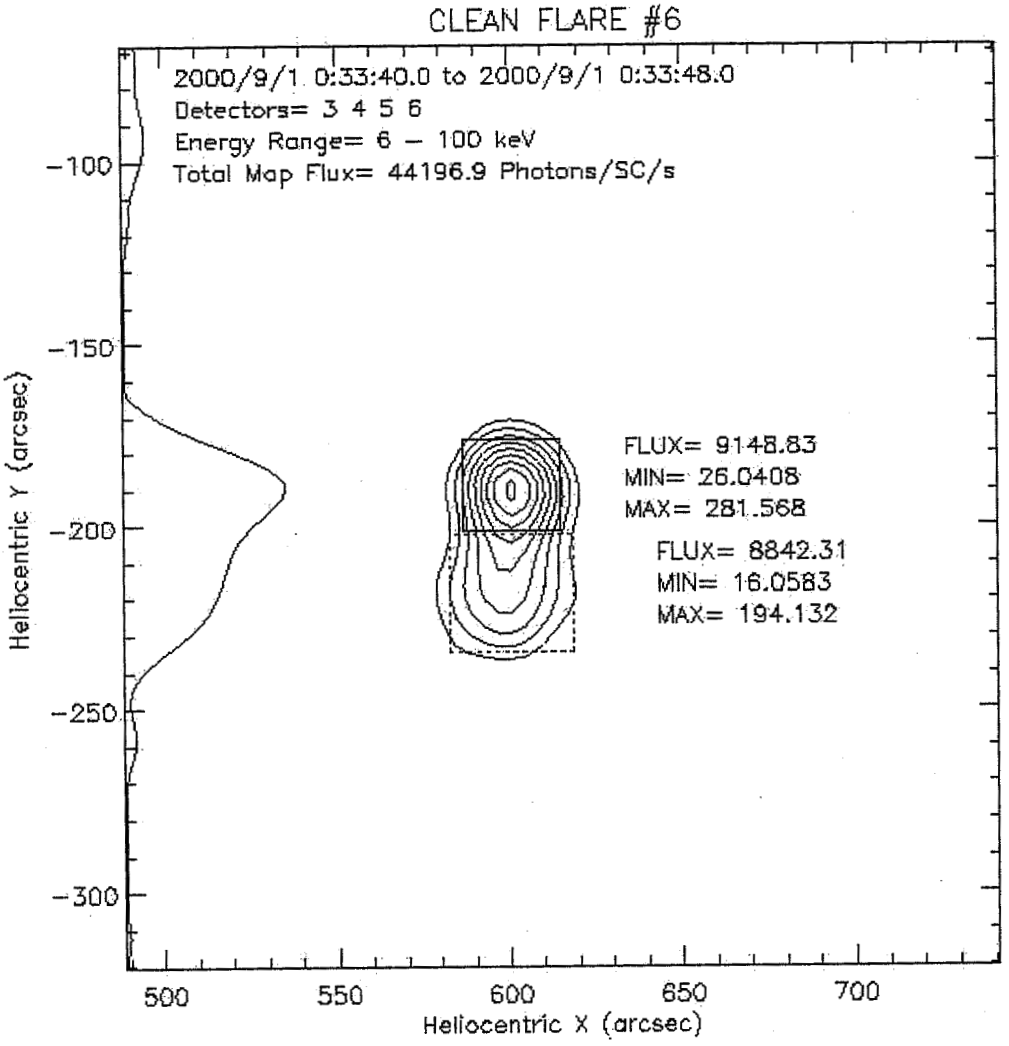
Flare 6 MEM_SATO

Ed Schmahl
Last modified: Wed May 23 09:15:47 EDT 2001

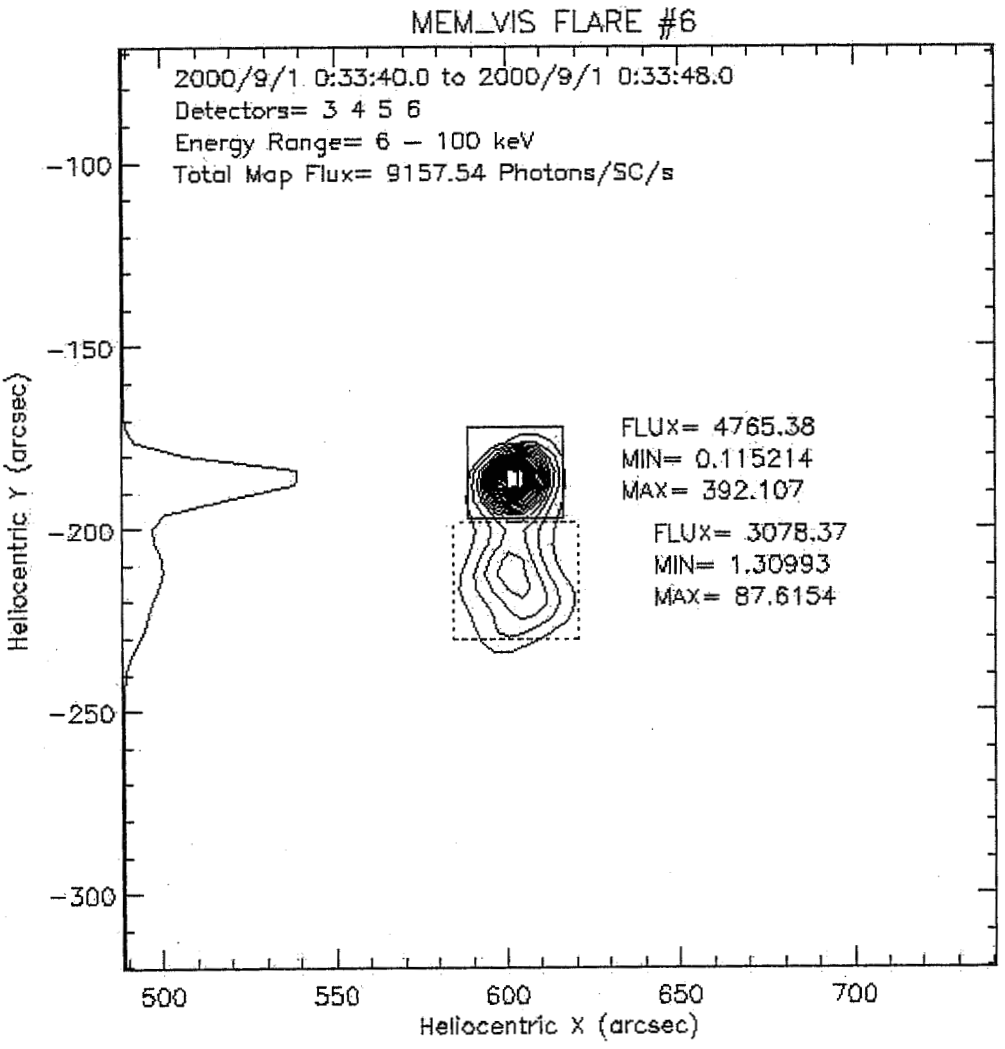
Simulated Flare #6 Reconstructions



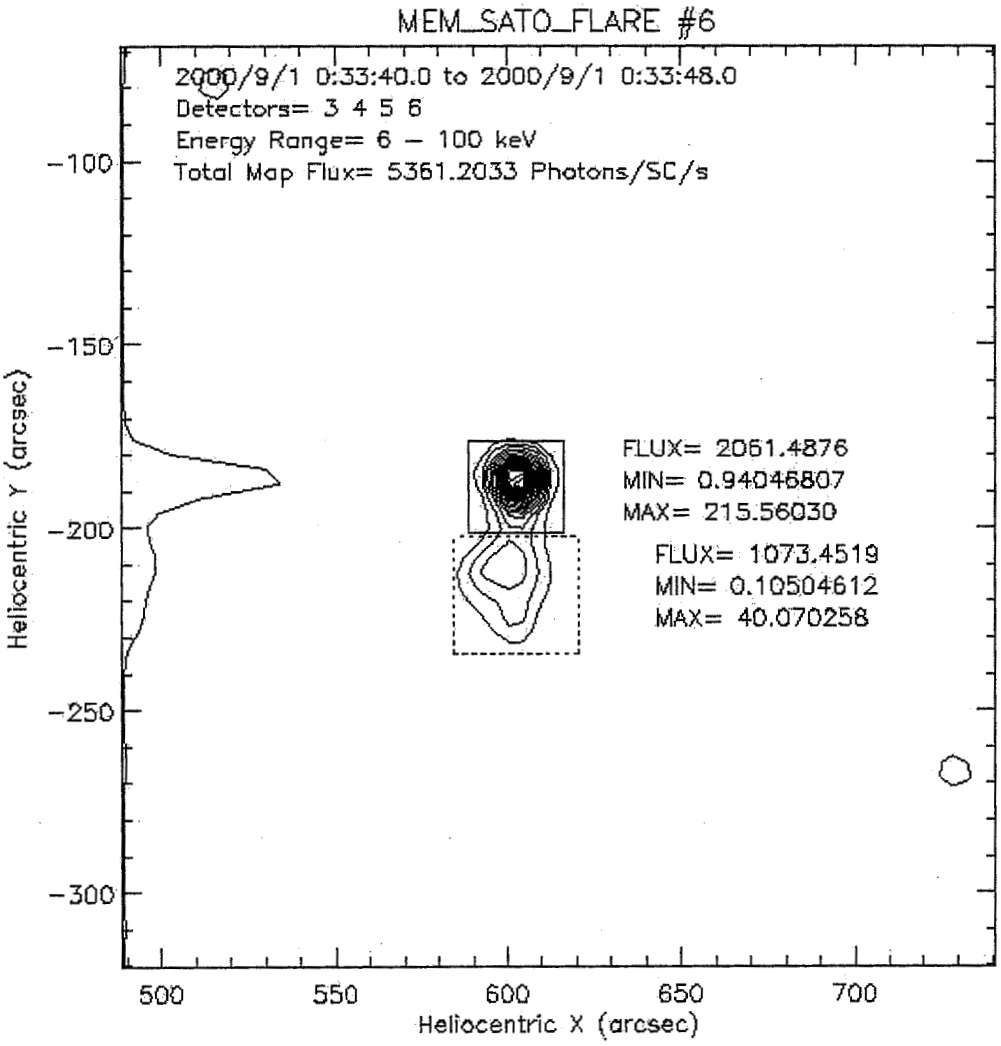
MODEL FLARE #6 (GH/JMcT)


CLEAN FLARE #6
2000/9/1 0:33:40.0 to 2000/9/1 0:33:48.0
Detectors= 3 4 5 6
Energy Range= 6 – 100 keV
Total Map Flux= 44196.9 Photons/SC/s
FLUX= 9148.83
MIN= 26.0408
MAX= 281.568
FLUX= 8842.31
MIN= 16.0583
MAX= 194.132
Heliocentric Y (arcsec)
-100
-150
-200
-250
-300
500
550
600
650
700
Heliocentric X (arcsec)

CLEAN


MEM_VIS FLARE #6
2000/9/1 0:33:40.0 to 2000/9/1 0:33:48.0
Detectors= 3 4 5 6
Energy Range= 6 – 100 keV
Total Map Flux= 9157.54 Photons/SC/s
FLUX= 4765.38
MIN= 0.115214
MAX= 392.107
FLUX= 3078.37
MIN= 1.30993
MAX= 87.6154
Heliocentric Y (arcsec)
-100
-150
-200
-250
-300
500
550
600
650
700
Heliocentric X (arcsec)

MEM_VIS


MEM_SATO_FLARE #6
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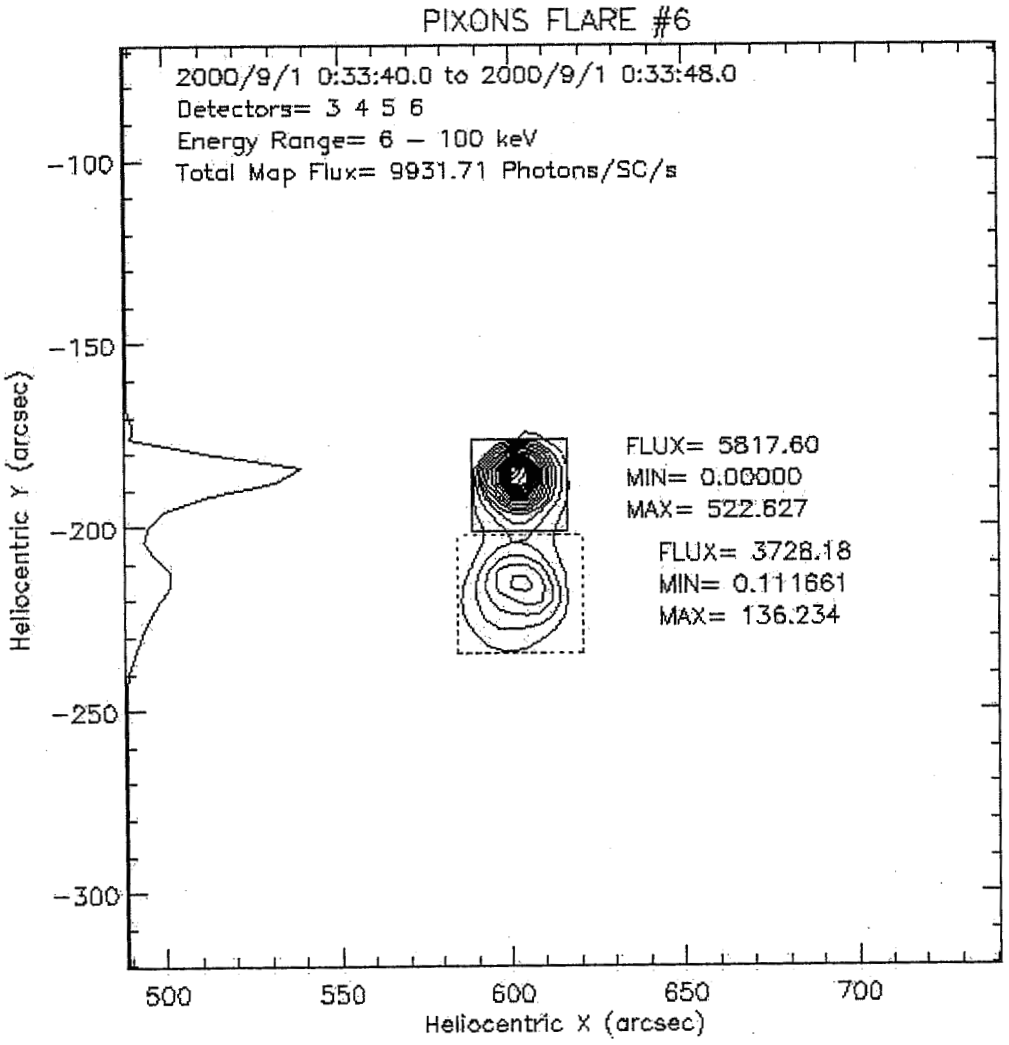
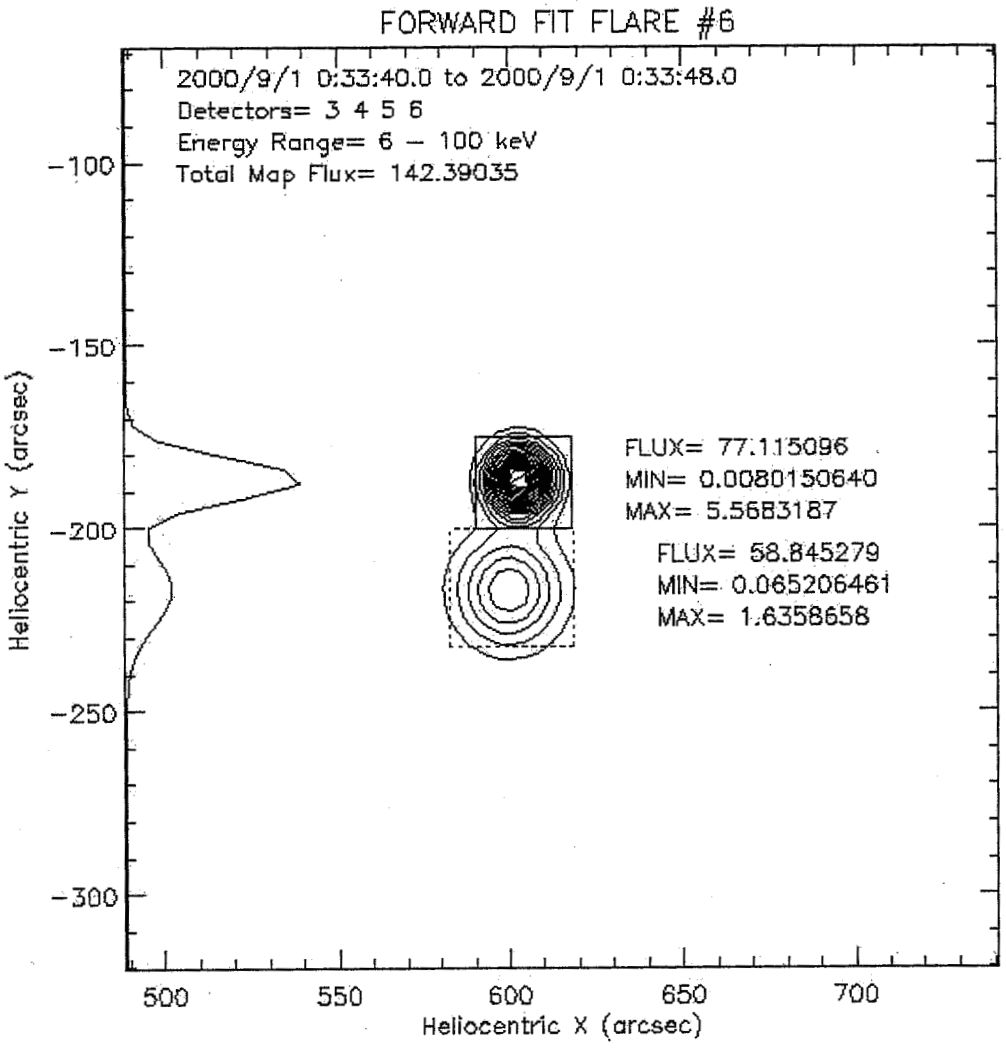
Detectors= 3 4 5 6
Energy Range= 6 – 100 keV
Total Map Flux= 5361.2033 Photons/SC/s
FLUX= 2061.4876
MIN= 0.94046807
MAX= 215.56030
FLUX= 1073.4519
MIN= 0.10504612
MAX= 40.070258
Heliocentric Y (arcsec)
-100
-150
-200
-250
-300
500
550
600
650
700
Heliocentric X (arcsec)

MEM_SATO


PIXONS FLARE #6
2000/9/1 0:33:40.0 to 2000/9/1 0:33:48.0
Detectors= 3 4 5 6
Energy Range= 6 – 100 keV
Total Map Flux= 9931.71 Photons/SC/s
FLUX= 5817.60
MIN= 0.00000
MAX= 522.627
FLUX= 3728.18
MIN= 0.111661
MAX= 136.234
Heliocentric Y (arcsec)
-100
-150
-200
-250
-300
500
550
600
650
700
Heliocentric X (arcsec)

PIXONS



FORWARD FIT

Ed Schmahl
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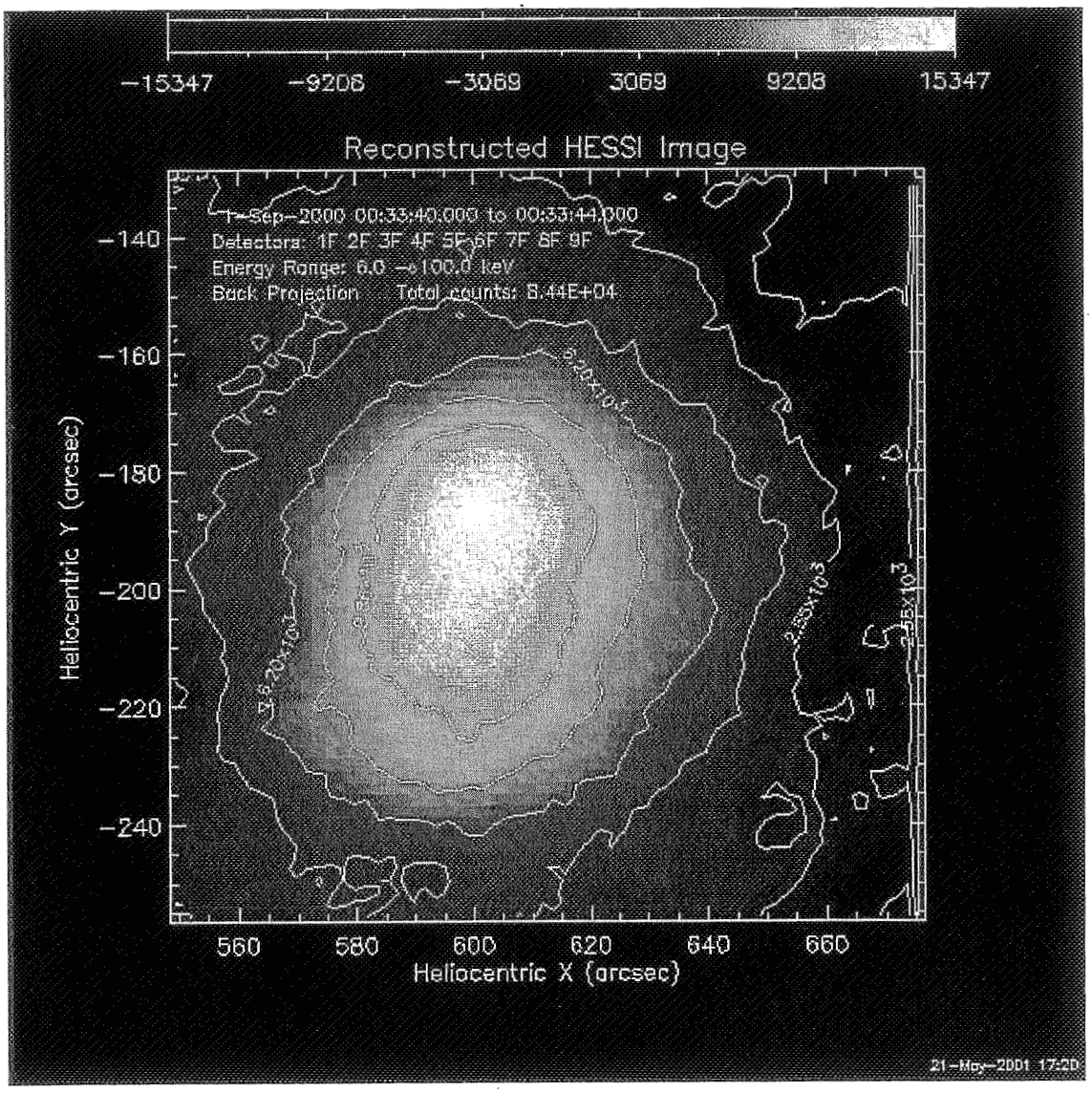
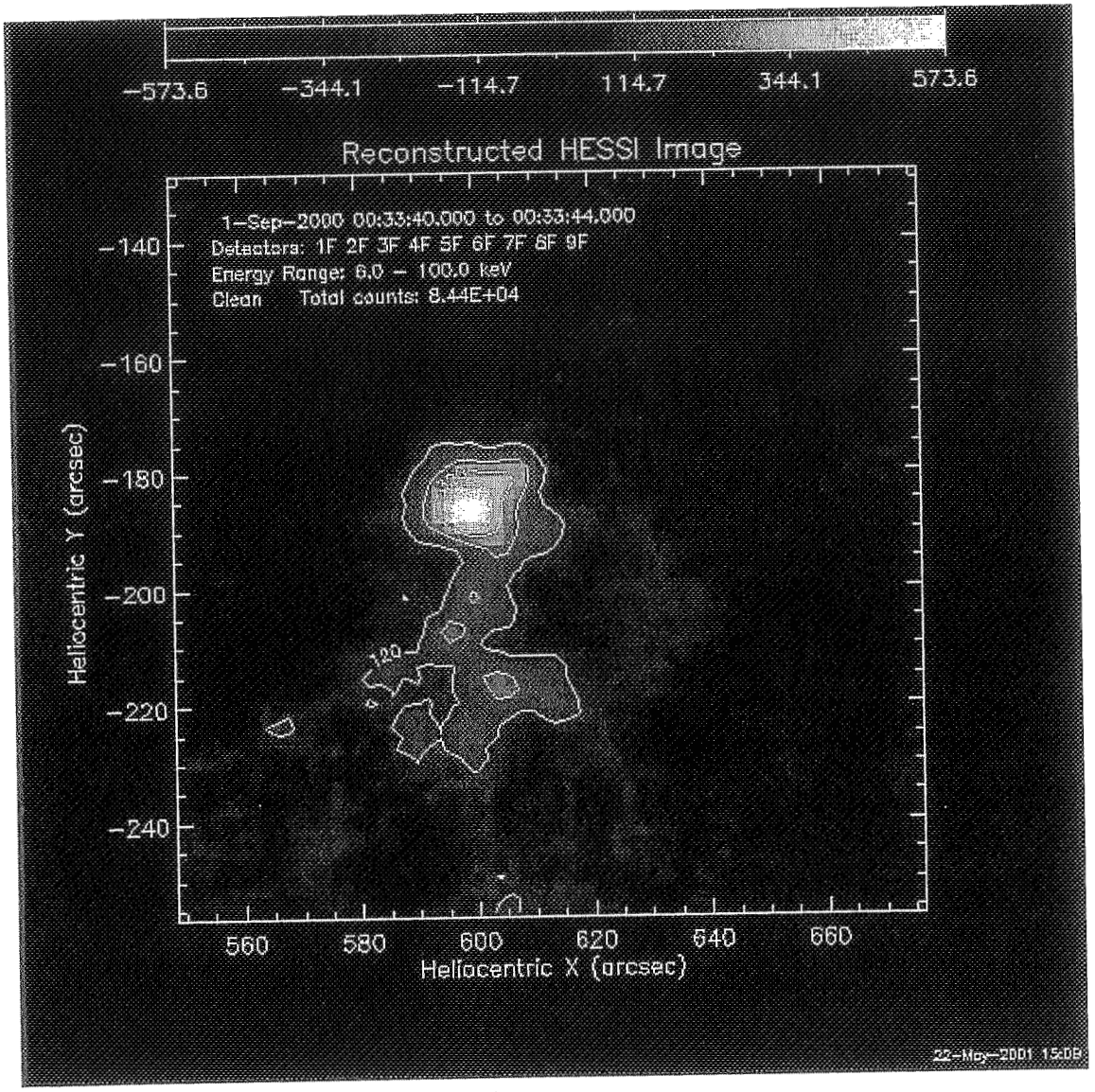
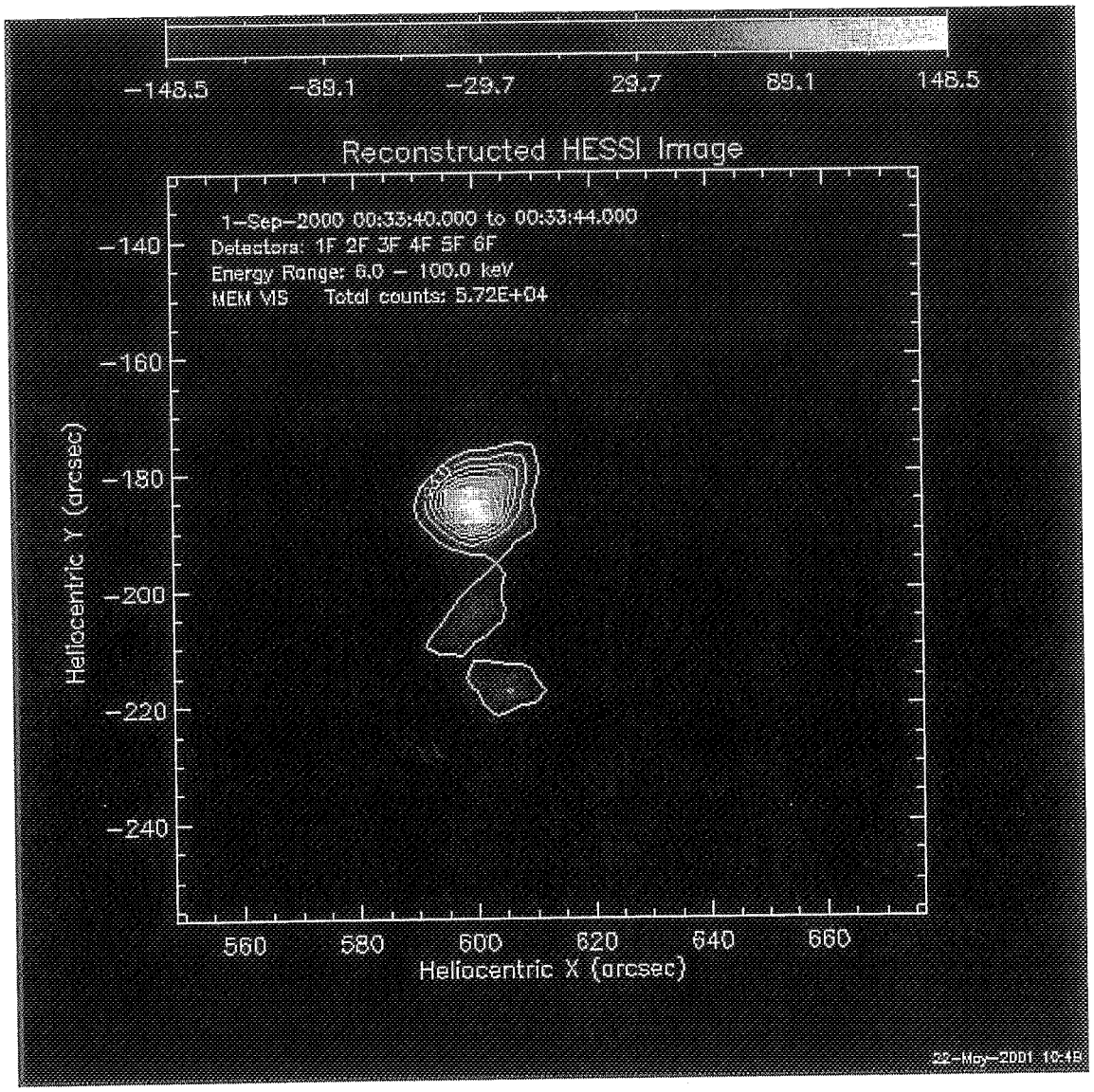
HESSI TEST-DATA RECONSTRUCTIONS (May 2001)



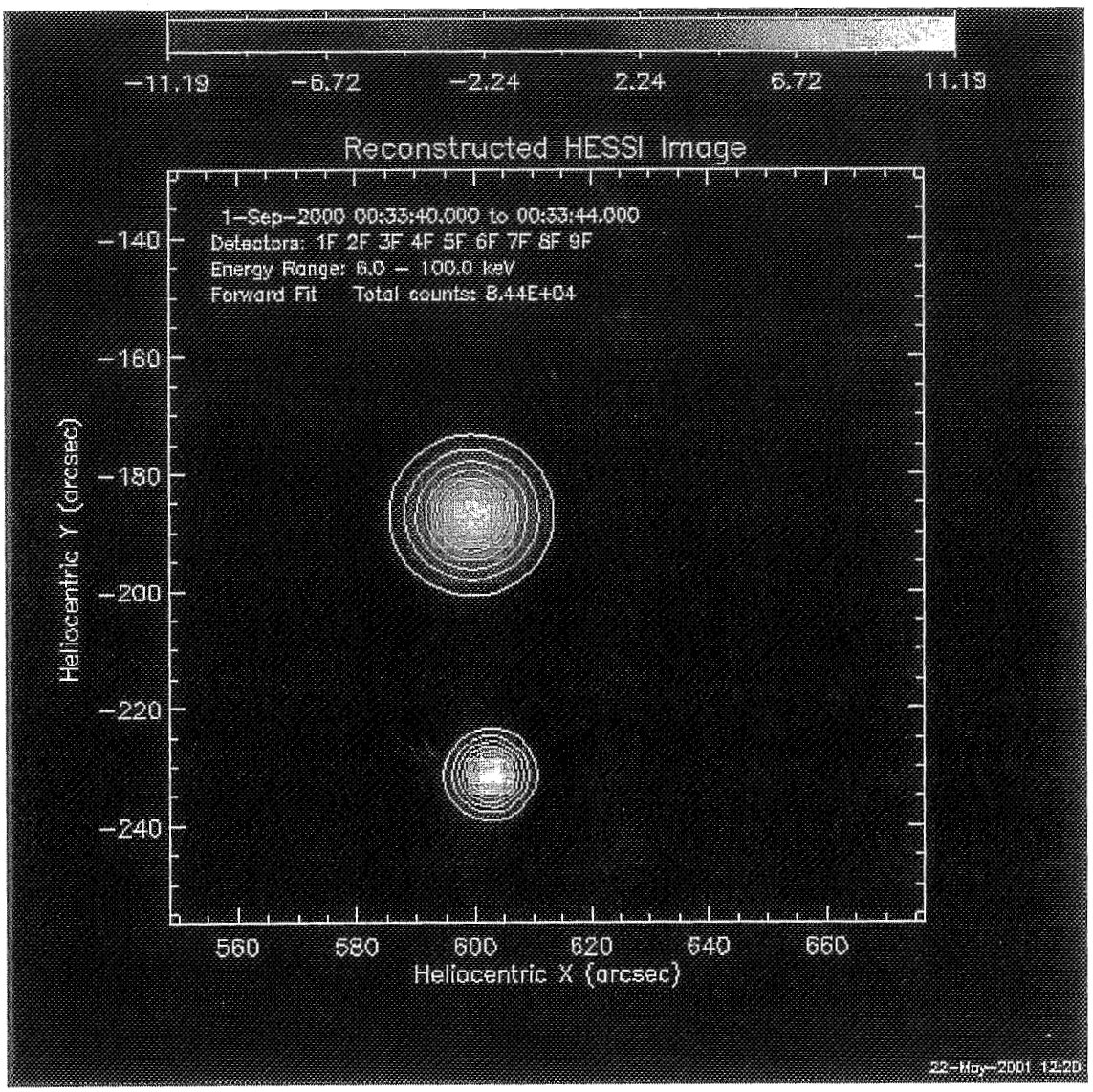
Flare 6 BACK PROJECTION



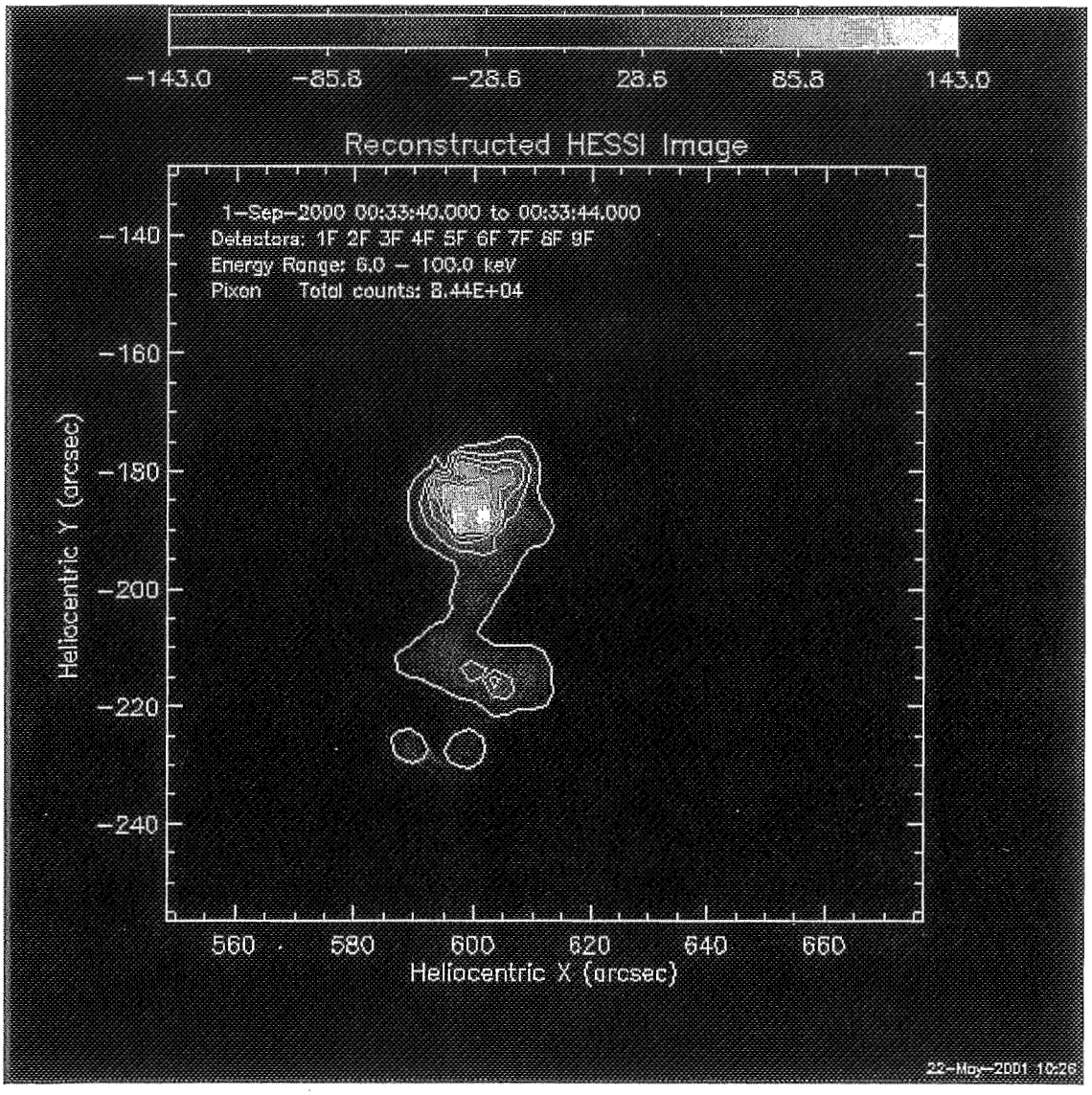
Flare 6 CLEAN



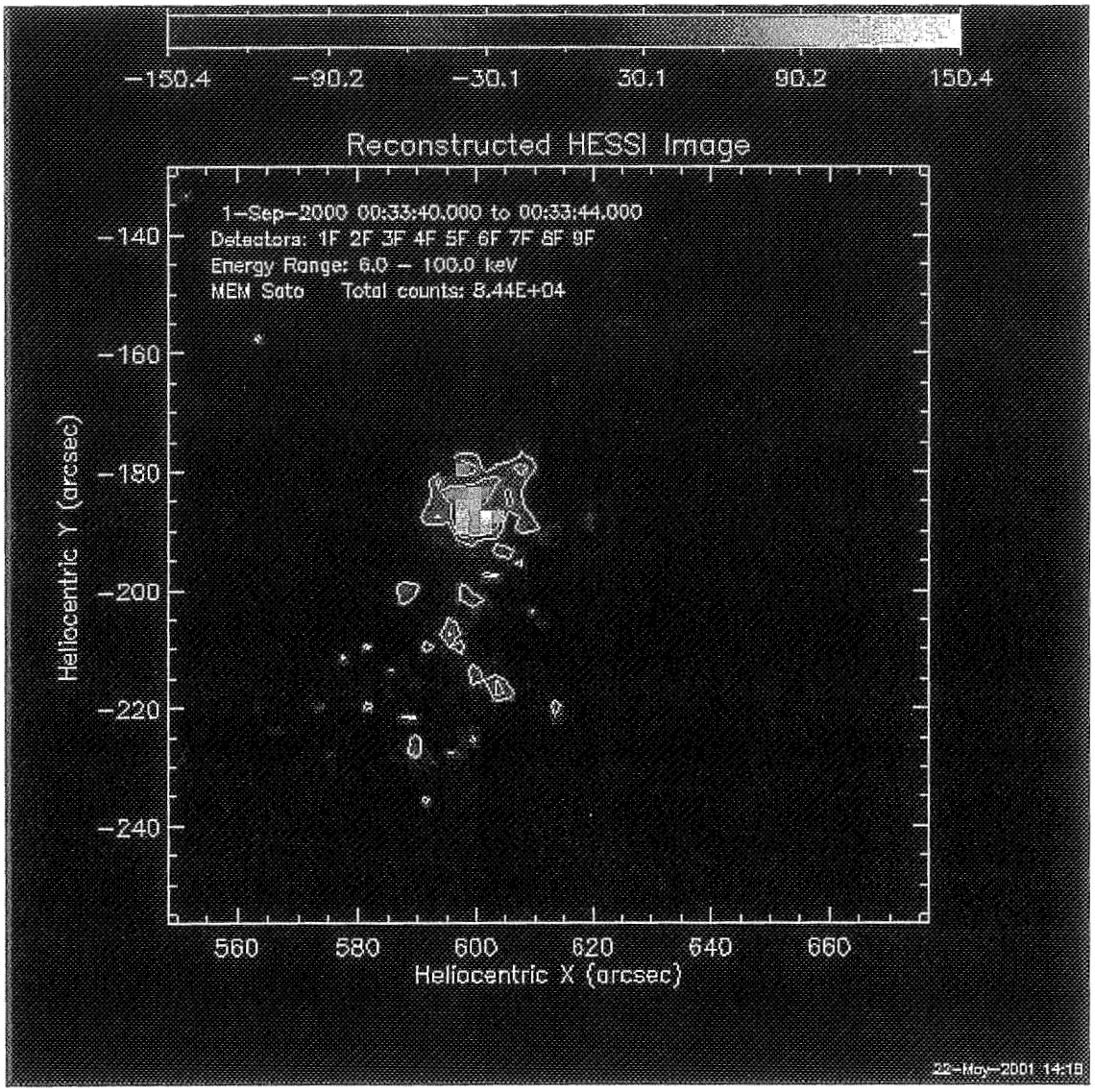
Flare 6 MEM_VIS



Flare 6 FORWARD FIT



Flare 6 PIXONS



Flare 6 MEM_SATO

Ed Schmahl
Last modified: Wed May 23 09:15:47 EDT 2001

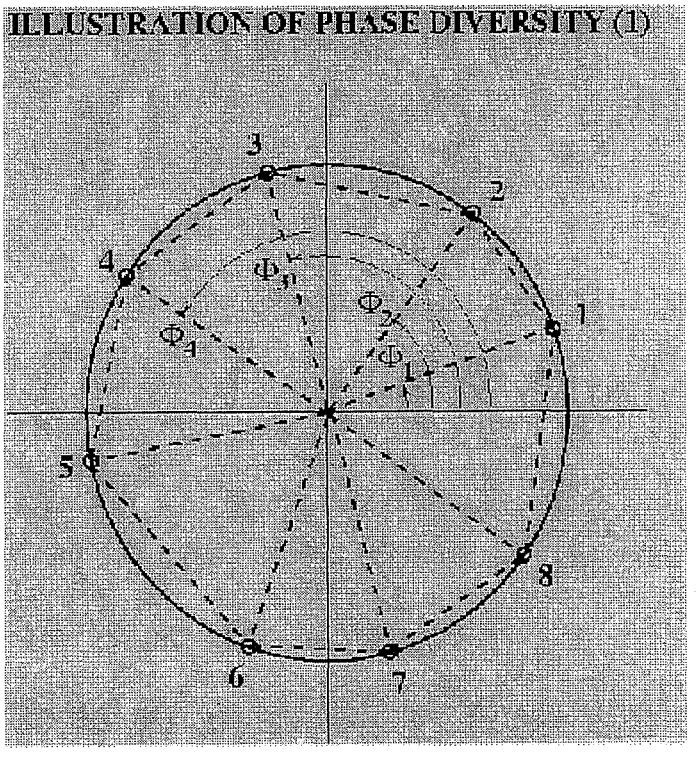
III. Measures of HESSI Phase Diversity (October 2001)

Measures of HESSI Phase Diversity

When computing visibility, or even making a Back-Projection map, it is important to have an optimum number of grid phases within each modulation cycle so that the image is properly sampled. If the spin is absolutely stable, this can be ensured by checking that the time bins are small enough that there are at least 4 in each cycle. The Nyquist theorem requires at least 2 samples per cycle, but 4 is the minimum for good fidelity.

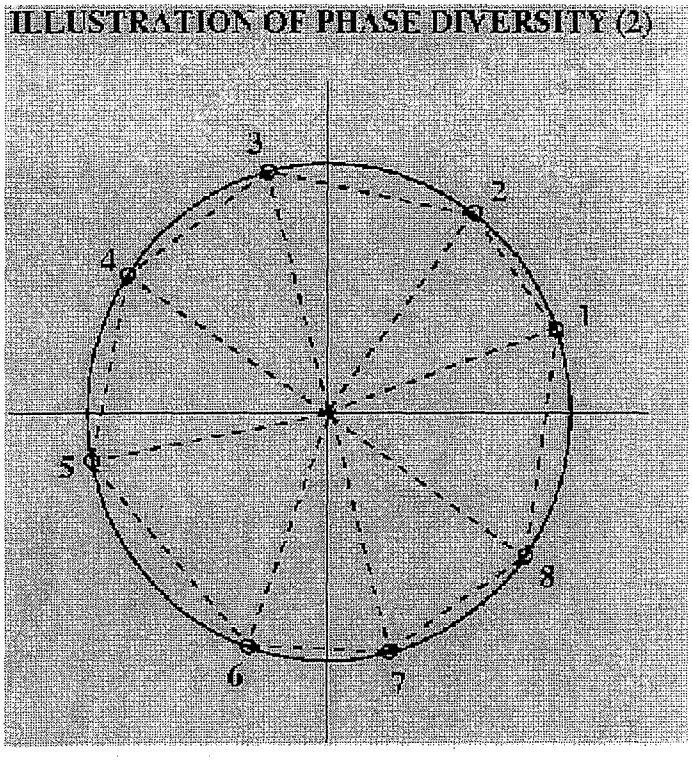
There are circumstances, however, when simply ensuring that there are 4 samples in each cycle is insufficient. This can happen when the pointing is changing at a rate comparable to the motion of the projected grid slats across map center on the Sun. This can result in the phases at map center not uniformly spanning 360 degrees within one cycle.

This is most easily illustrated by plotting the phases as azimuths in the phase plane. (See Figure)

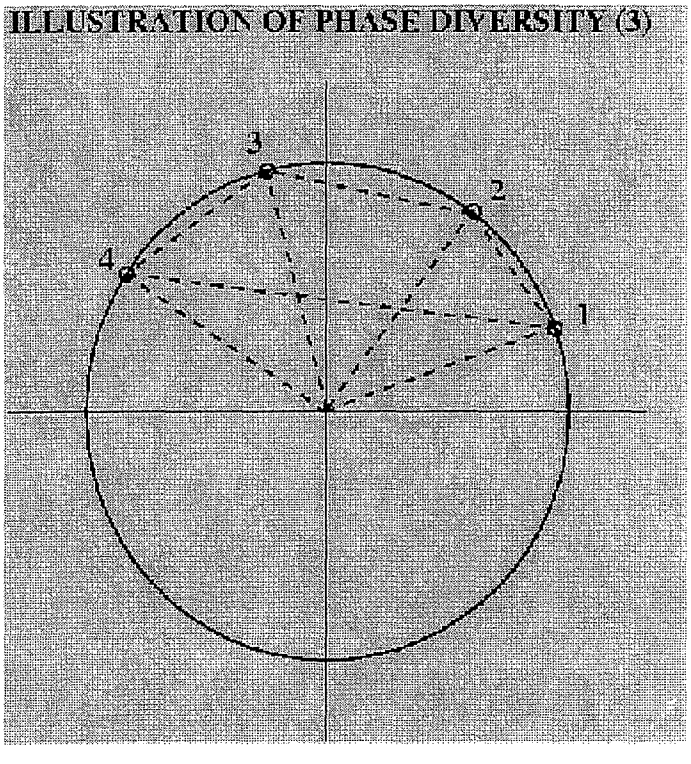


The phases in this example are represented as azimuth angles for 8 points on a circle. The first 4 phases are indicated by arcs measured from the positive x axis. Clearly, the 8 phases are spread widely enough apart that they would allow a good fit to a sinusoid representing a modulation cycle. The first 4, which span less than 180 degrees, would permit a determination of a sinusoid from the countrates in the cycle, but it would be less well determined.

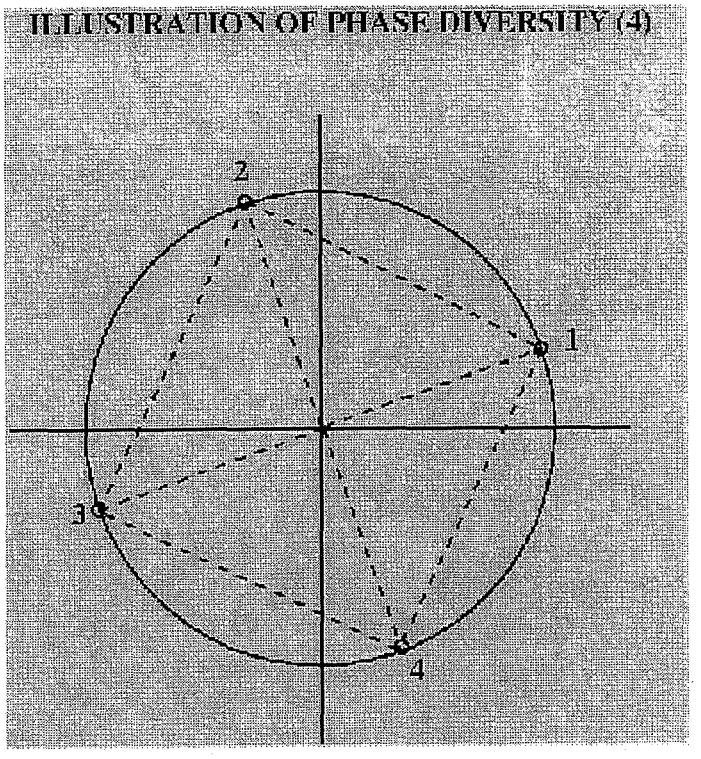
What is a quantitative measure of the amount of "phase diversity" within a cycle? A simple measure can be created graphically from the diagram above. If the small circles at the ends of the radial lines are joined, in the example shown, 8 triangles are formed:



If, as is true here, the phase angles form a nearly regular figure, the phase diversity is large, and the sum of the areas of the included triangles is close to the maximum possible (the area of a regular octagon, slightly bigger than this slightly irregular octagon .)



If, however, the cycle is poorly sampled, as shown here for 4 phases, then the total area is less than (about half) the maximum possible, i.e., 2, which is the area of the square inscribed in the unit circle shown in the next figure:



Here the phases are optimally distributed for 4 samples, and the measure of phase diversity is a maximum.

The expression that gives the ratio of the total area of the triangles associated with N arbitrary phases in one cycle to the maximum possible area is :

$$D = \frac{\sum_{i=1}^{N} sin|\Phi_j - \Phi_{j+1}|}{N sin(2\pi/N)}$$

This function has a minimum value of 0 (worst phase diversity) and a maximum value of 1 (best phase diversity). It has been coded in the IDL function HSI_PHZ_DIVERSITY.PRO. (Care has been taken to ensure that the phases are sorted after taking them modulo 2 pi.)

```
function hsi_phz_diversity,phz,VERB=verb
;+
```

```
; PURPOSE:
; To estimate the diversity of a phase vector.
; Returns normed sum of the areas of triangles formed by  successive
; phasors exp(i*phz).

; INPUTS:
;     phz = vector of phases (domain: -infinity to +infinity)
;     phz must be sorted in increasing order.  If it is not, then
;        it will be sorted.
;     The phases will be wrapped modulo 2*!pi if max-min GT 2*!pi
;     If VERB is set, informational messages will be printed
; OUTPUTS:
;     diversity (SCALAR)
;
; EXAMPLE:
;  phz=20.+2*!pi*randomu(seed,100)
;  print,hsi_phz_diversity(phz,/verb)
;
; METHOD:
;  Computes sum of the areas of triangles formed by  successive phasor
;    if this is large, the phases are diverse
;    if it is small, the phases are not diverse
;  For N phasors, the largest possible area is (N/2)*sin(2*!pi/N)
;  So the sum is normalized to this value.
;  Good phase diversity occurs when diversity is almost 1.0
;  If the phasors are spread out over
;                                    1 quadrant,  diversity=0.5
;                                    2 quadrants, diversity=0.707
;                                    3 quadrants, diversity=0.79
;
; HISTORY:
;  EJS 10/24/01  schmahl@hessi.gsfc.nasa.gov
;-

N=n_elements(phz)
twopi=2*!pi

if max(phz)-min(phz) GT twopi then  begin
  if keyword_set(verb) then message,'Wrapping phases to 2*!pi',/info
  M=1+long(-(min(phz)<0)/twopi)   ; Number of negative multiples of two
  phz=(M*twopi+phz) mod twopi     ; add enough multiples of twopi befor
endif

w=where(phz[0:N-2] GT phz[1:N-1],nw) ; if phz is already sorted, nw=0
if nw eq 0 then begin
  f=phz
endif else begin
  if keyword_set(verb) then message,'Sorting phz.',/info
  f=phz(sort(phz))
endelse
```

```
area=0.

for j=0,N-2 do area=area+abs(sin((f[j]-f[j+1])))/2.   ; add each triang
area=area+abs(sin((f[N-1]-f[0])))/2   ; add last area

diversity=area/((N/2.)*sin(twopi/N))   ; normalize to maximum possible
if diversity GT 1.0 then message,'ERROR: Phase diversity GT 1'

RETURN,diversity

END
```

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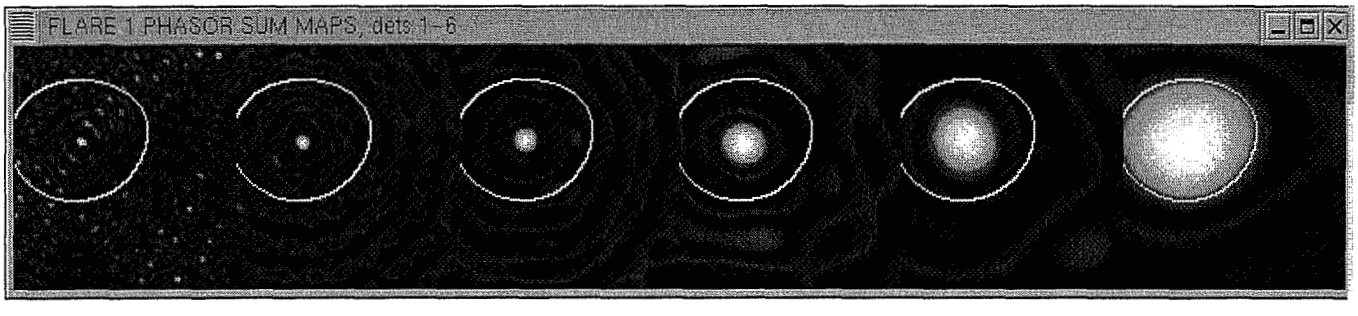
IV. Over Resolution Evaluation (August 2001)

Tests of *hsi_overres_eval.pro* using flare testdata

```
;+
FUNCTION HSI_OVERRES_EVAL,CBE,NX,DMAX,ftmap,NODISPLAY=nodisplay,$
                          PIXEL_SIZE=pixel_size,VERBOSE=verbose,  $
                          STATS=stats,TITLE=title
; PURPOSE:
;   Tries to evaluate a HESSI calibrated event list to determine which
;   subcollimators over resolve the source.
;
; METHOD:
;   Starts with detector dmax, makes a Fourier transform map
;   (hsi_phasor_sum), selects the half-power contour of the source,
;   then goes to the finer subcollimators, making FT maps and
;   determining the ratio of the maximum inside the contour
;   to the maximum outside the contour.
;   Usually, if this ratio is less than one, the subcollimator
;   overresolves the source, but noise, erronious pointing, or other
;   factors can throw it off.  Therefore the default is to show the
;   single-collimator maps.
```

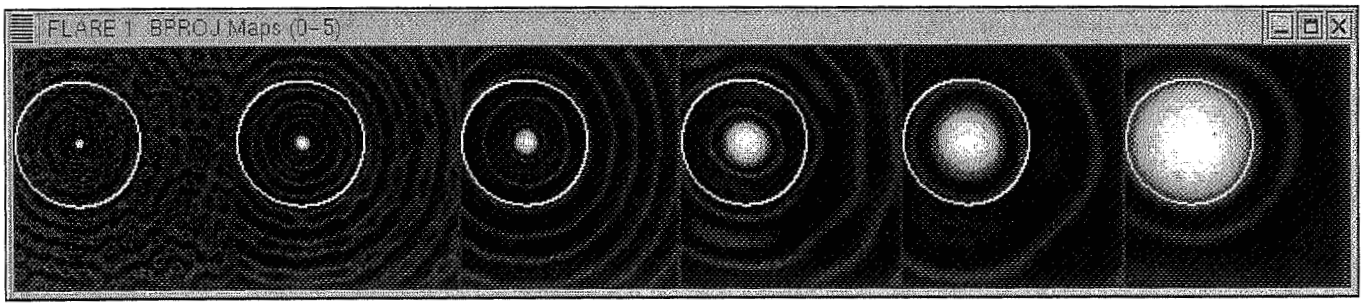
Examples for testdata flares 1-9

FLARE 1 Single-Collimator FT Maps



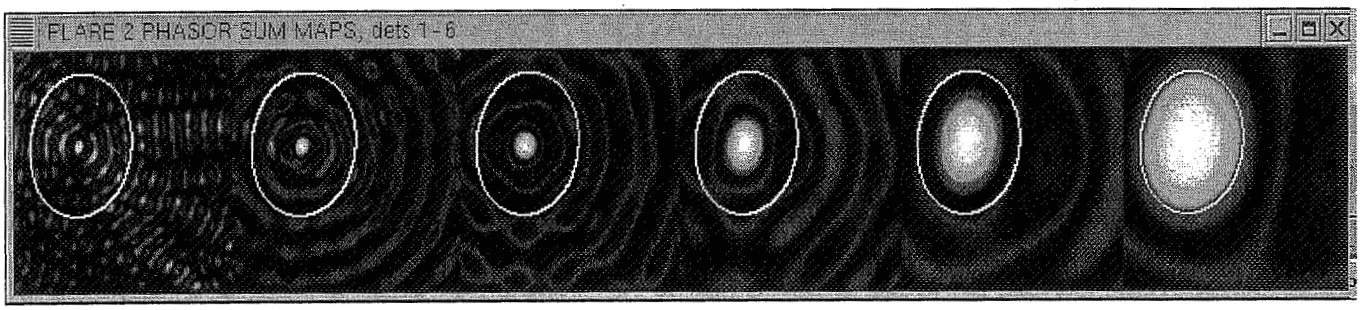

```
Output of hsi_overres_eval:
FLARE 1 PHASOR SUM MAPS, dets 1-6 INSIDE/OUTSIDE RATIO OF MAXIMA
       1      1.33643
       2      1.97347
       3      2.37062
       4      2.05639
       5      2.11248
       6      1.90198
COMMENT:
This flare is a point source, and the program correctly finds a ratio > 1
for all single-collimator maps.

Compare the above with the results of Back projection:
```




```
INSIDE/OUTSIDE RATIO OF MAXIMA
    1       2.31373
    2       3.23918
    3       3.34304
    4       2.47523
    5       2.89087
    6       1.90433
```

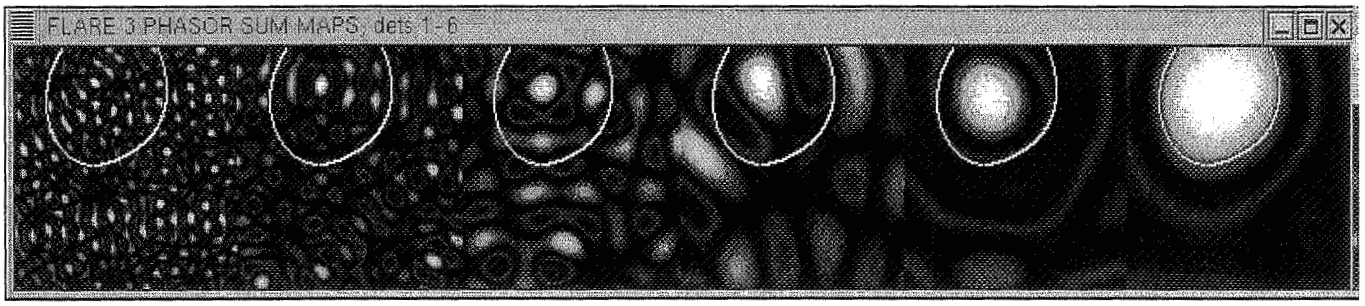
FLARE 2 Single-Collimator FT Maps



Output of hsi_overres_eval:

```
FLARE 2 PHASOR SUM MAPS, dets 1-6 INSIDE/OUTSIDE RATIO OF MAXIMA
       1       1.67966
       2       2.28242
       3       2.36724
       4       2.36474
       5       2.35560
       6       1.91350
COMMENT:
This flare is also a point source, and the program correctly finds a ratio > 1
for all single-collimator maps.
```

FLARE 3 Single-Collimator FT Maps

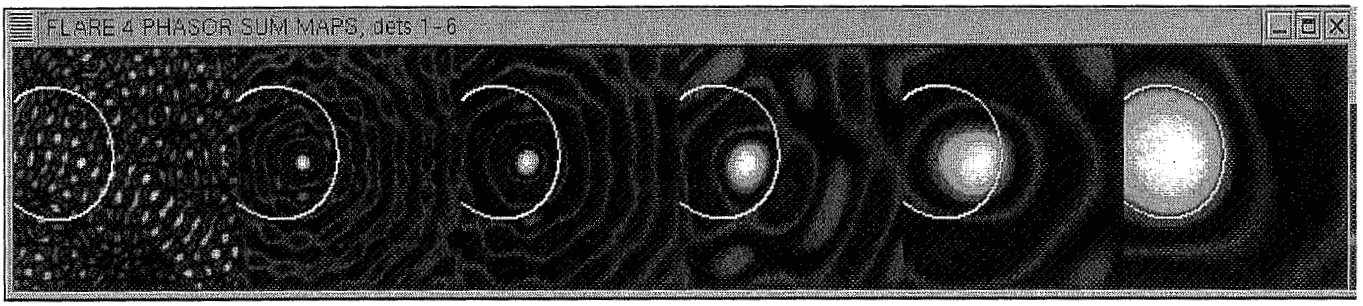


Output of hsi_overres_eval:

```
FLARE 3 PHASOR SUM MAPS, dets 1-6 INSIDE/OUTSIDE RATIO OF MAXIMA
       1      1.08606
       2      1.26030
       3      1.30649
       4      1.38147
       5      2.07780
       6      1.24848
```

COMMENT:
This flare is a weaker point source, and the program correctly finds a ratio > 1 for all single-collimator maps.

FLARE 4 Single-Collimator FT Maps

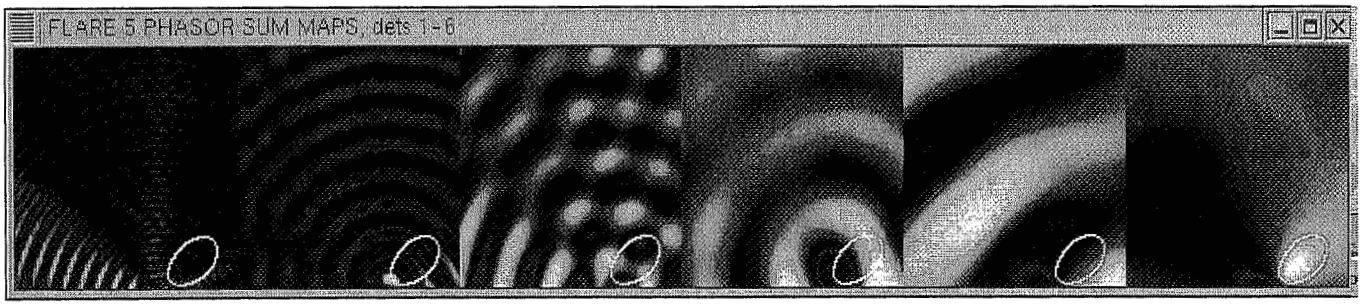


Output of hsi_overres_eval:

```
FLARE 4 PHASOR SUM MAPS, dets 1-6 INSIDE/OUTSIDE RATIO OF MAXIMA
       1     0.936529
       2      2.56635
       3      2.13025
       4      2.01656
       5      1.40083
       6      1.88308
```

COMMENT:
Also a point source, but for some reason the coarsest map shows an offset of the source from that of the finer-collimator maps. (The cause is being investigated.)

FLARE 5 Single-Collimator FT Maps



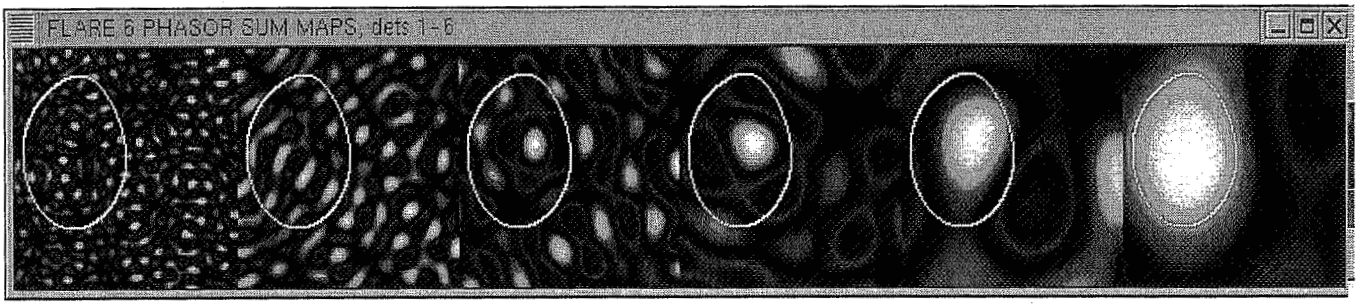
Output of hsi_overres_eval:

FLARE 5 PHASOR SUM MAPS, dets 1-6 INSIDE/OUTSIDE RATIO OF MAXIMA

1	0.278255
2	1.02931
3	0.710092
4	0.736607
5	0.362777
6	1.95015

COMMENTS:
This flare has only a 0.5-second duration, so all the maps are poor. Nothing should be expected from hsi_overres_eval in this case.

FLARE 6 Single-Collimator FT Maps



Output of hsi_overres_eval:

FLARE 6 PHASOR SUM MAPS, dets 1-6 INSIDE/OUTSIDE RATIO OF MAXIMA

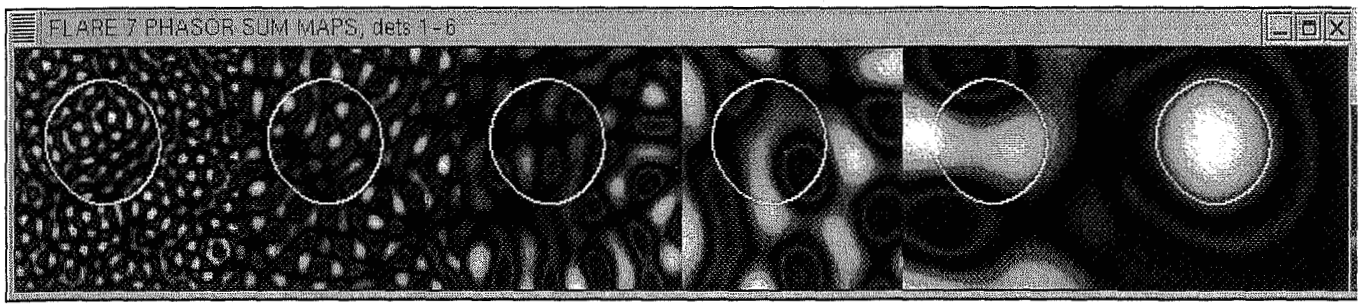
1	0.816310
2	0.887275
3	1.06506
4	1.46256
5	1.14944
6	1.88405

COMMENTS:

This is a double source where the components have sizes of 5 and 10 arcsec, so collimators 1 and 2 should over resolve both

components. The outputs succesfully indicate this.

FLARE 7 Single-Collimator FT Maps



Output of hsi_overres_eval:

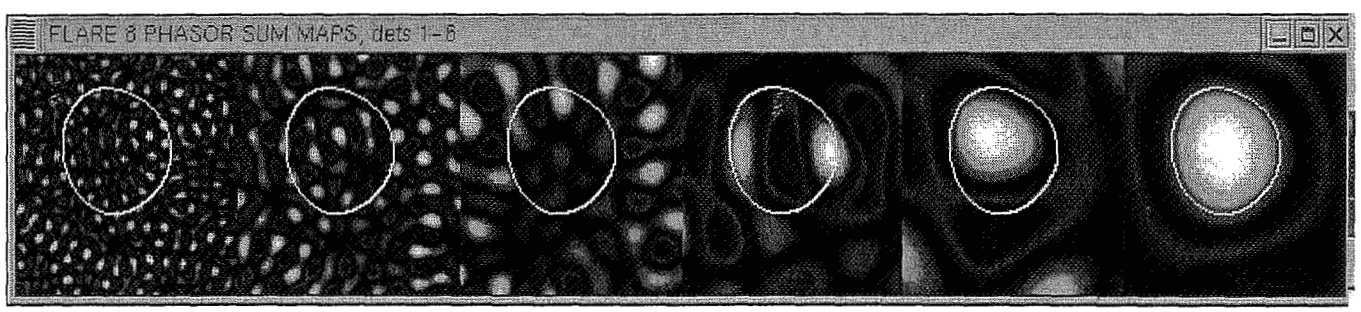
```
FLARE 7 PHASOR SUM MAPS, dets 1-6 INSIDE/OUTSIDE RATIO OF MAXIMA
       1      1.06818
       2     0.936754
       3     0.477178
       4     0.723433
       5     0.939746
       6      1.91462
```

COMMENTS:
This flare has two 4 x 20 arcsec components, and should be over resolved by detectors 1,2,3 and 4, but the output shows ratios of 1.06 and 0.94 for detectors 1 and 2. These values are clearly wrong. Looking at the maps for collimators 1 and 2, there doesn't seem to be anything of significance in the contours.

This is an example of how the human eye can make a better determination than a programmed single-number statistic.

Does anyone have a suggestion for how a program could make a better assessment?

FLARE 8 Single-Collimator FT Maps



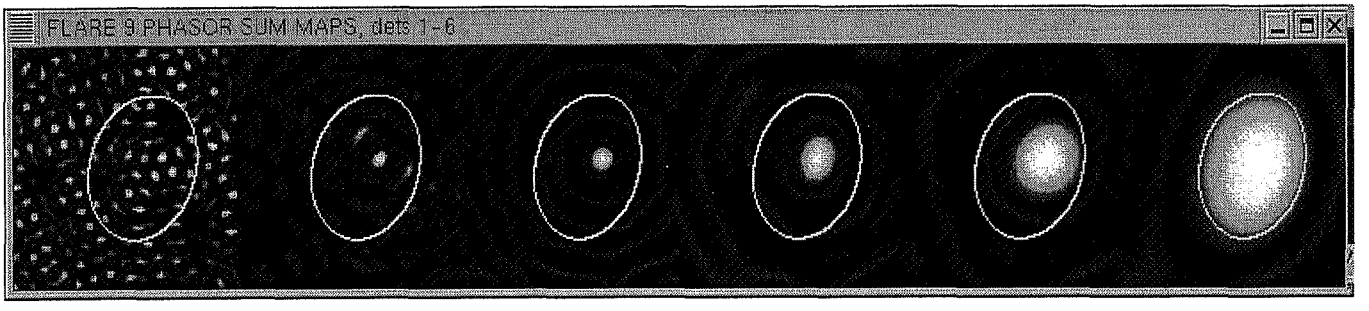
Output of hsi_overres_eval:

```
FLARE 8 PHASOR SUM MAPS, dets 1-6 INSIDE/OUTSIDE RATIO OF MAXIMA
       1      0.934892
       2       1.06527
       3      0.824776
       4       1.10071
       5       1.41920
       6       1.90997
```

COMMENTS:
This flare is two 5-arcsec Gaussians. The output rightfully finds collimators 4, 5 and 6 not over resolving, but says that collimator 2 is also not overresolving (ratio=1.06527).

Again, the single-ratio statistic fails, and a better statistic is needed.

FLARE 9 Single-Collimator FT Maps



Output of hsi_overres_eval:

```
FLARE 9 PHASOR SUM MAPS, dets 1-6 INSIDE/OUTSIDE RATIO OF MAXIMA
       1       1.06612
       2       1.98707
       3       2.89170
       4       2.24095
       5       2.26897
       6       1.88859
```

COMMENTS:
This is a pair of 1-arcsec Gaussians, and, as desired, the program finds them not over resolved by all the collimators.

The program and included functions:

HSI_OVERRES_EVAL.PRO